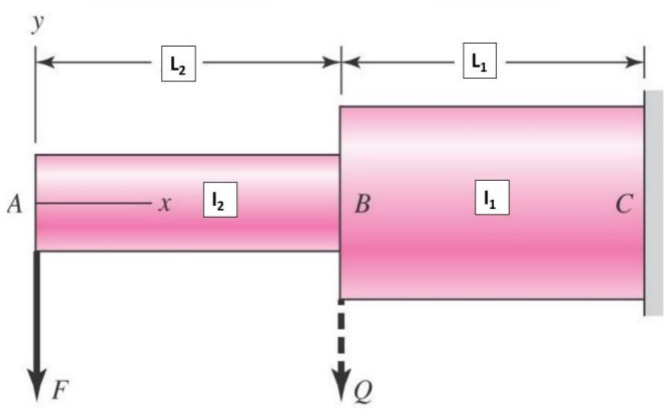
FEA Module 1 - Beam Deflection

Cases



Case 1: $I_2 = 2I_1$
Case 2: $I_2 = 4I_1$
Case 3: $L_2 = 3L_1$

Results

Table 1: Beam Deflection at points A and B

Case	Analytic A (mm)	FEA A (mm)	Analytic B (mm)	FEA B (mm)
1	3.46E-3	3.57E-3	0.96E-3	1.01E-3
2	2.11E-3	2.29E-3	0.48E-3	0.51E-3
3	3.12E-3	3.19E-3	1.95E-3	2.00E-3

Analysis

For case 1, the deflection found in the FEA analysis was relatively close to that of the analytical solution(3% error for point A and 5% error for point B) . For case 2, like case 1, the deflection found in the FEA analysis was relatively close to that of the analytical solution (8% error for point A and 6% error for point B). For case 3, like the previous cases, the deflection found in the FEA analysis was relatively close to that of the analytical solution (2% error for point A and 3% error for point B). Overall, the FEA simulation performed quite well. It gave results that were reasonably close to the analytical solution.

Analytical Solutions

Case 1

Define the load q and moment m:

$$q = -F + R < x - l >, m = -F < x >$$

As $I_2 = 1I_2$, the moment m can be defined as:

$$m = -\frac{F < x >}{I_1} + \frac{F < x - \frac{l}{2} >}{I_1} + \frac{FL < x - \frac{l}{2} >^0}{4I_1} - \frac{< x - \frac{l}{2} >}{2I_1}$$

Taking the integral:

$$\theta \cdot E = -\frac{F < x >^2}{2I_1} + \frac{F < x - \frac{l}{2} >^2}{2I_1} + \frac{FL < x - \frac{l}{2} >}{4I_1} - \frac{< x - \frac{l}{2} >^2}{4I_1} + C_1$$

B.C. $\theta = 0$ at x = l:

$$C_1 = \frac{5L^2}{16I_1}$$

Finding deflection :

$$\delta \cdot E = -\frac{F < x >^3}{6I_1} + \frac{F < x - \frac{l}{2} >^3}{6I_1} + \frac{FL < x - \frac{l}{2} >^2}{8I_1} - \frac{< x - \frac{l}{2} >^3}{12I_1} + \frac{5L^2}{16I_1} < x > + C_2$$

B.C. $\delta = 0$ at x = l:

$$C_2 = -\frac{3FL^3}{16I_1}$$

Point A:

$$\delta|_{x=0} = -\frac{3FL^3}{16EI_1}$$

Point B:

$$\delta|_{x=l} = -\frac{5FL^3}{96EI_1}$$

Case 2

Define the load q and moment m:

$$q = -F + R < x - l >, m = -F < x >$$

As $I_2 = 4I_1$, the moment m can be defined as:

$$m = -\frac{F < x >}{I_1} + \frac{F < x - \frac{l}{2} >}{I_1} + \frac{FL < x - \frac{l}{2} >^0}{8I_1} - \frac{< x - \frac{l}{2} >}{4I_1}$$

Taking the integral:

$$\theta \cdot E = -\frac{F < x >^2}{2I_1} + \frac{F < x - \frac{l}{2} >^2}{2I_1} + \frac{FL < x - \frac{l}{2} >}{8I_1} - \frac{< x - \frac{l}{2} >^2}{8I_1} + C_1$$

B.C. $\theta = 0$ at x = l:

$$C_1 = \frac{7L^2}{32I_1}$$

Finding deflection :

$$\delta \cdot E = -\frac{F < x >^3}{6I_1} + \frac{F < x - \frac{l}{2} >^3}{6I_1} + \frac{FL < x - \frac{l}{2} >^2}{16I_1} - \frac{< x - \frac{l}{2} >^3}{24I_1} + \frac{7L^2}{32I_1} < x > + C_2$$

B.C. $\delta = 0$ at x = l:

$$C_2 = -\frac{11FL^3}{96I_1}$$

Point A:

$$\delta|_{x=0} = -\frac{11FL^3}{96EI_1}$$

Point B:

$$\delta|_{x=l} = -\frac{5FL^3}{192EI_1}$$

Case 3

The moment m can be defined as:

$$m = -\frac{F < x >}{I_1} + \frac{F < x - \frac{l}{4} >}{I_1} + \frac{FL < x - \frac{l}{4} >^0}{8I_1} - \frac{< x - \frac{l}{4} >}{2I_1}$$

Taking the integral:

$$\theta \cdot E = -\frac{F < x >^2}{2I_1} + \frac{F < x - \frac{l}{4} >^2}{2I_1} + \frac{FL < x - \frac{l}{4} >}{8I_1} - \frac{< x - \frac{l}{4} >^2}{4I_1} + C_1$$

B.C. $\theta = 0$ at x = l:

$$C_1 = \frac{17L^2}{64I_1}$$

Finding deflection :

$$\delta \cdot E = -\frac{F < x >^3}{6I_1} + \frac{F < x - \frac{l}{4} >^3}{6I_1} + \frac{FL < x - \frac{l}{4} >^2}{16I_1} - \frac{< x - \frac{l}{4} >^3}{12I_1} + \frac{17L^2}{64I_1} < x > + C_2$$

B.C. $\delta = 0$ at x = l:

$$C_2 = -\frac{65FL^3}{384I_1}$$

Point A:

$$\delta|_{x=0} = -\frac{65FL^3}{384EI_1}$$

Point B:

$$\delta|_{x=l} = -\frac{27FL^3}{256EI_1}$$